

THE INFORMATION PROCESS RELATIONSHIP IN MATHEMATICAL  
METAPHYSICS: THE PROBLEM OF RANDOMNESS IN THE LOGIC OF  
ARITHMETIC

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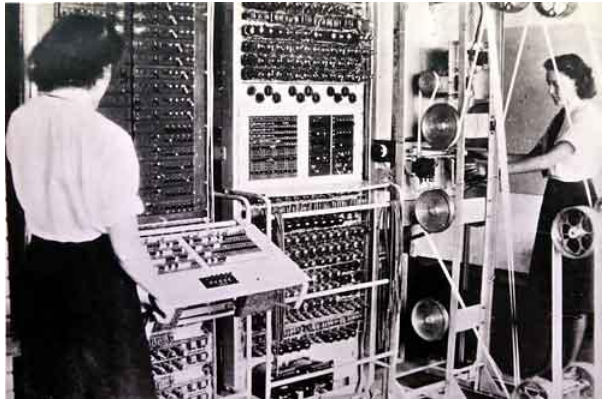
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## Abstract

This paper will discuss information theorist Gregory Chaitin's claim that randomness is a characteristic of arithmetic. The discussion strongly suggests the need for a rethinking of metaphysics. I will follow the problem of algebraic countability from Hilbert, Gödel and Turing and end with Gregory Chaitin's demonstration of randomness in arithmetic. The significant claim that randomness is a characteristic of attempting to answer some important but very unusual questions in the logic of arithmetic will be examined in light of the metaphysical implications. If one can find an issue in the system of arithmetic then this means that the same issues will permeate in other similar systems. Chaitin claims that a primacy of information is necessary for an appropriate understanding of the issue of randomness in arithmetic which leads me to the speculation that a new type of information process metaphysics is the key to bridging process philosophy with a modern mathematical-scientific understanding of the world.

# THE PROCESS INFORMATION RELATIONSHIP IN MATHEMATICAL METAPHYSICS: THE PROBLEM OF RANDOMNESS IN THE LOGIC OF ARITHMETIC



The Colossus, electronic digital computer, UK 1943.<sup>1</sup>

This paper will discuss information theorist Gregory Chaitin's idea that a primacy of information is necessary for an appropriate understanding of the issue of randomness in arithmetic. We will follow the issues of completeness, countability and randomness from Hilbert, Gödel and Turing leading to Chaitin's demonstration. The significant claim is that characteristic randomness implies a basic instability in the foundations of metaphysics. I hope to show that information, process and randomness are crucial points within the argument and that this in turn exhibits the need for more exploration in the foundations of metaphysics. Perhaps a new type of information process metaphysics is the key to bridging process philosophy with a modern mathematical-scientific understanding of the world.

## Hilbert's Problem

I should say first of all, this: that it shall be possible to establish the correctness of the solution by means of a finite number of steps based upon a finite number of hypothesis which are implied in the statement of the problem

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<sup>1</sup> The first computer was named Colossus (1943), not to be confused with the 1966 Eniac computer. Colossus was a secret enterprise of the World War II western coalition, located at Bletchley Park („Station X”). Image from [http://en.wikipedia.org/wiki/Colossus\\_computer](http://en.wikipedia.org/wiki/Colossus_computer)

and which must always be exactly formulated. This requirement of logical deduction by means of a finite number of processes is simply the requirement of rigor in reasoning. David Hilbert, 1900<sup>2</sup>

The year is 1900. The place is Paris. David Hilbert is addressing the International Congress of Mathematicians. Hilbert outlines 23 problems that remained to be adequately addressed by mathematicians. He threw down the gauntlet to the congress and urged them to fix mathematics truly and finally. Within the text of the second problem, “The Compatibility of the Arithmetical Axioms”, Hilbert stated the common faith of mathematicians and scientists at the beginning of the twentieth century; A faith that has not, to a large extent, diminished in one hundred years since his address.

I am convinced that it must be possible to find a direct proof for the compatibility of the arithmetical axioms, by means of a careful study and suitable modification of the known methods of reasoning in the theory of irrational numbers.<sup>3</sup>

According to Hilbert, a systematic search for meaningful statements in mathematics or well formed formulae and their proof in mathematics is a pursuit that is both possible and realistic. And, as Nagel and Newman state, “The first step in the construction of an absolute proof [of mathematical statements], as Hilbert conceived the matter, is the *complete formalization* of a deduction system.”<sup>4</sup>

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<sup>2</sup> David Hilbert, “Mathematical Problems”, lecture delivered before the International Congress of Mathematicians at Paris in 1900, translated by Mary Winton Newson for Bulletin of the American Mathematical Society 8 (1902) 437-479. I am quoting from a text version of the paper and page numbering will vary slightly. p. 439

<sup>3</sup> Hilbert, p. 443.

<sup>4</sup> Ernest Nagel and James R. Newman, *Gödel's Proof*, revised edition, (New York University Press: New York, 2001) p. 25.

To achieve complete formalization of a formal axiomatic system such as arithmetic, one is first concerned with logical consistency and completeness of the system. The system is composed of axioms, logical operators and syntactic rules for proposition production. We refer to any proposition that abides by the syntactic rules as a well formed formula.

In a consistent system, it would not be possible to prove both a well formed formula and its negation. Thus, no contradictory statements within the formal system should be possible. In other words, one cannot generate the falsity or  $F_0$ . This is important since the existence of the falsity would mean that any statement, any well formed formula, could be proven true.<sup>5</sup>

Completeness is also important. Completeness means that all well formed formula in the system can either be proved or disproved.<sup>6</sup> For arithmetic, this means any arithmetic theorem can be proved or disproved based on the axioms using the formal rules of inference of the system. Consequently, Hilbert's challenge can be interpreted as a completeness and consistency check of the system. The work one needs to do is to prove or disprove all possible well formed formulae.

Bertrand Russell and Alfred North Whitehead attempted to formalize arithmetic by starting with the axioms of arithmetic and then generating, through proof, all of mathematical theory. No small job. This is only slightly less work than generating and checking all possible proofs but at least it became a problem of progressive proof rather than one of universal checking. Only valid theorems would be attended to, and then demonstrated, and all the invalid ones would be thus avoided.

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<sup>5</sup> For any well formed formula  $P$ ,  $F_0 \rightarrow P$  is always true.  $F_0 \rightarrow P$ , is really  $F \rightarrow (T \vee F) \rightarrow T$ , i.e. true for  $P$  or not  $P$ .

<sup>6</sup> Recall that a proved statement is called a theorem.

An additional outcome would be the construction of the foundation of mathematical theory.

Using a particular set of axioms of arithmetic that seemed promising, commonly labeled as Peano's axioms of arithmetic, Russell and Whitehead proceeded to construct set theory and number theory. The result is the monumental (but incomplete) three-volume tome *Principia Mathematica*<sup>7</sup> spanning thousands of pages of small print of axioms followed by proof and theorems. Never is doubt was the assumption that such a system would necessarily be consistent and that one would, in due course, prove everything known in mathematics, and perhaps much more. It was a smug, stiff and arrogant self-assurance set fully in the tradition of Logicism<sup>8</sup> and Laplace's assumption.

### Gödel's Incompleteness Theorem

Thirty-one years after Hilbert's speech in Paris, the logicist program is suddenly set back by an unsuspected and unusual theorem developed by Kurt Gödel the paper *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*.<sup>9</sup> Gödel's Incompleteness Theorem states that "If you assume a system is consistent, then it must be incomplete" in its proof, a somewhat involved set of self-references, conclusively shows that the completeness of the axioms of arithmetic can never be

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<sup>7</sup> Bertrand Russell and Alfred North Whitehead, *Principia Mathematica*, (Cambridge Press, Cambridge, 1989) Using Peano's Axioms Russell and Whitehead take almost 70 pages just to get  $1+1 = 2$ ! A fourth volume on geometry was planned but never completed.

<sup>8</sup> Logicism: The school of thought that assumes that one could reduce all of mathematics to logic is referred to, alternatively, as positivism, logical positivism or logicism.

<sup>9</sup> Kurt Gödel, *Ueber formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme [On Formally Undecidable Propositions of Principia Mathematica and Related Systems – 1931]*, translated by B. Meltzer, (Dover Publications: 1992).

proved. A simple enough theorem to understand, but it meant that at least part of Hilbert's conviction was wrong; no search program can show a logical system to be both consistent and complete.

You can not provide proof of all possible valid theorems without assuming some truth from the start. This result would shake the very foundations of mathematics and its impact is still being felt in the academics of mathematics today. It goes without saying that Russell and Whitehead never did finish the 4<sup>th</sup> volume of *Principia*.

### **Turning-Chaitin Uncomputability**

Incompleteness is only part of the problem. Recall the approach suggested by Hilbert's challenge in terms of the analogy of the algorithmic program as stated by Chaitin. In 1934 Alan Turing was working on some bothersome problems in number theory and developing some of the first code for computers. Turing asked the question, "Can we truly show that arithmetic is both consistent and complete?" In part, Turing needed to deal with Hilbert's original problem. But, unlike Russell and Whitehead, Turing defined the problem as an algorithmic search for **information** about the well formed formulae. Turing thought the requirement of consistency and completeness should be approached by what Hilbert called the *ent-schei-dung*-problem or the decision problem.

Solving the decision problem for a formal axiomatic system is giving an algorithm that enables you to decide whether any given meaningful assertion is a theorem or not. A solution to the problem is called a decision procedure.<sup>10</sup>

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<sup>10</sup> Gregory J. Chaitin, *The Limits of Mathematics* (Singapore: Springer-Verlag, 1998), pp. 3-4.

Gödel assumed that the system was consistent, but incomplete. With this decision algorithm, we could ignore incompleteness but still attempt to show consistency. Logicism still had a hope, although this was not Turing intention.

Turing demonstrated that the resolution of a fully demonstrable logical system would hang on the idea of computability. A computable algorithm is one for which there is a method to calculate or compute the algorithm to a result, to the point where information results. The assumption for computability is that the algorithm(ic) (program) for a particular theorem will **halt** (or stop or complete) itself in a finite time of processing. If the program halts the problem is computable, or, alternatively, the process is said to be denumerable or countable. Using a Cantor diagonal argument, one can check if the problem is always denumerable, i.e. the well formed formulas can always be counted.<sup>11</sup> The question is “Does the program work?” What happens if the program fails to print a line or fails to compute a result? What if the program fails to stop or halt?

No surprise, Turing quickly proved that Hilbert's problem was uncomputable. There is no algorithm, no mechanical procedure, which will decide if the  $n$ th computer program ever outputs an  $n$ th result.<sup>12</sup>

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<sup>11</sup> Although possibility infinite. By countable here we mean that a one-to-one correspondence with the set of natural numbers is possible. The set of rational numbers, all the natural numbers and fractions, anything that is writable as  $p/q$ , where  $p, q$  are both integers and  $q$  not zero; this set is countable by Cantor's argument. (Diagonalization Theorem). The real numbers is the set of rational number union with the set of irrational numbers like  $\pi$ , square root of 2, non-repeating, non-terminating decimal representations. The real numbers fail the Diagonalization Theorem and are not countable. Thus, there are at least two types of numerical infinity: countable and uncountable. Finite sets, by the way, are countable by definition.

<sup>12</sup> Chaitin actually describes this in terms of as a numerical process and translates the algorithmic issue as a numerical process. The output is the  $n$ th digit.

## David Hilbert Was Not a Twit<sup>13</sup>

I would like to point out an important shift in thinking here. Turing is after **information about the process** of running through a complex algorithm; a result in the decision problem. In other words, if you put the algorithm to the test and you run through all the well formed formulae, you become **informed** in terms of yes or no, whether the well formed formula is a theorem. You gain information about the well formed formula.

In his paper “Randomness in Arithmetic and the Decline and Fall of Reductionism in Pure Mathematics,”<sup>14</sup> Chaitin, following Turing, suggests that we can formally express Hilbert’s search as a simple algorithm that can be processed by a computer program. An algorithmic search program much like the original one expressed by Hilbert almost 100 years previously and analyzed by Turing more than 70 years previously. But today, we have the actual possibility of really running through the algorithm that generates all possible proofs based on the axiom set and check for the occurrence of both  $A$  and  $\text{not } A$ . The charm of Chaitin’s approach is to use contemporary algorithmic coding and processing with the latest electronic-silicon marvel and let the beast go. Of course, Chaitin found that the algorithm will fail to halt. Chaitin has made this code available and his demonstration is relatively reproducible.<sup>15</sup>

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<sup>13</sup> Gregory Chaitin, “The Decline and Fall of Reductionism in Pure Mathematics”, in *The Limits of Mathematics*, (Springer-Verlag: Singapore, 1998)

<sup>14</sup> Chaitin, “Randomness,” pp. 314-328. Lecture given 22 October 1991.

<sup>15</sup> I lack the expertise in information theory required to compress and fully qualify the results but I have yet to find dissenters in the literature, which, is as one imagines, rather small. Steven Wolfram, a “super expert” programmer and creator of Mathematica, a computer algebra system used by professional mathematicians, agrees with Wolfram’s result. If anyone has the expertise in this field, Wolfram is it.

As a corollary result, finding that the algorithm is uncomputable means that the system is incomplete, since if the decision procedure fails to halt some theorems cannot be proved. Still nothing earth shattering considering.

Much more interesting is the provision of inherent randomness, a feature I have previously shown<sup>16</sup> to be a key indication of the need for process thinking at the foundational level of metaphysics.<sup>17</sup> Chaitin's coding of the basic Hilbert algorithm has the outcome of inherent **randomness** in number theory<sup>18</sup>, the basic theory of addition and subtraction of numbers.

The halting probability is an expression of the probability that the algorithmic search will halt. Chaitin computes the halting probability and develops its expression as  $\sum_{p \text{ halts}} 2^{-|p|}$ . It turns out that converting into a binary string; a representation composed of only 1s and 0s requires as much information as the original number. If you have N bits of information for , then the representation requires N bits, and you cannot reduce or compress the information contained in the expression. We say that this kind of information is irreducible. In addition, Chaitin discovered that if you were to examine any particular digit of , you would find that the probability of the digit being a 1 or 2 or 3 ... or 9, would be equally 10%. Chaitin assembles a

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<sup>16</sup> See J.V. Marc Corbeil, *Process Environmental Philosophy*, [online cited 13 July 2006] <http://www.mcorbeil.com>.

<sup>17</sup> I lack the formal expertise in information theory required to compress and fully qualify the results but I have yet to find dissenters in the literature, which, is as one imagines, rather small. Steven Wolfram, a “super expert” programmer and creator of Mathematica, a computer algebra system used by professional mathematicians, agrees with Wolfram's result. If anyone has the expertise in this field, Wolfram is it.

<sup>18</sup> This is worse than randomness in arithmetic.

Diophantine<sup>19</sup> equation such that the equation “has a finite number of solutions if a particular individual bit of  $\pi$  is a 0, and it has infinite number of solutions if that bit is a 1.”<sup>20</sup> Thus, to decide if the Diophantine equation has either finite or infinite solutions is the same as trying to compute a particular digit of  $\pi$ , which has characteristic randomness.

Let us recap. Hilbert suggested a problem; all theorems in arithmetic are provable using an axiomatic system that is consistent and complete. Gödel proved the system could not be complete. Turing proved Hilbert's original algorithmic search program could be represented algorithmically, but the system is essentially uncomputable, in addition to being incomplete. Chaitin demonstrated that the halting program related to algorithmic coding of Hilbert's problem can be expressed by the halting probability, and this reveals characteristic randomness in arithmetic.

This well and truly means that Hilbert's problem is ended. The pursuit of an algorithmic search for well formed formulae leads to results as good as flipping a coin to decide the truth of any particular theorem.

It is no surprise to me that Chaitin found randomness in the structure of mathematics since randomness has caused so much issue in so many other areas and subjects. But consider the facts. Chaitin's focus on the proof structure as a computational representation of the logical structure of Hilbert's problem puts the randomness at the very foundation of mathematics. The whole mess pivots on

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<sup>19</sup> This is making use of Hilbert's tenth question involving Diophantine equations in elementary number theory. The equation  $ax + by = c$  is a simple linear Diophantine equation. Given  $a, b, c$  one can solve this equation for a particular solution or a general solution  $(x,y)$ .

<sup>20</sup> Chaitin, *Loss*, p. 20.

knowing about a particular bit of **information** at a particular level of consideration of a complex system. Sounds all too familiar to me!

We are suggesting that complexity, randomness, information and process are all fundamental features of reality. And, this is demonstrably necessary as per the logical structure of mathematics. How important is this? What answers might such thinking provide?

Consider the following statement by John A. Jungerman. “Countervailing the general tendency of the universe toward increased entropy, as specified by the second law of thermodynamics, is the order and decreased entropy produced by complex systems.”

Jungerman is talking about the complex system of evolution on earth. Randomness must be a characteristic of reality; otherwise the order that results from evolution would not be possible according to thermodynamic theory. But we cannot yet account for the randomness. But, suppose that we can use the inherent characteristic of randomness in mathematics to **increase the** randomness in a numerical string, centered on processes of information within a complex system. This is essentially providing a source of randomness; an explanation for decreased points of entropy in evolutionary theory.

This thinking behind this paper is in reference to A.N. Whitehead and work that has evolved from his ideas of process. Whitehead considered philosophy and adventure in ideas ... speculations that should not be limited. We also are aware of the fact that Whitehead was only deeply exposed to the first stage of Quantum mechanics. He never really seemed to incorporate the second wave, pardon the pun. Since then, we have had superstring, supersymmetry and a whole range of

mathematics related to complexity theory called chaos theory. I would like to end this talk with a speculation, perhaps one that some of us might find uncomfortable.

The development of thought seems to have historically gone from basic philosophy to mathematics to reduction transformations to metaphysics and then to physics. The progress of physics has gone from pre-Newtonian, Newtonian, Electromagnetism, Thermodynamics, Relativity, Quantum I and II, Superstring, Supersymmetry and now, who knows what.

Each step or revolution in physics can be associated with a reduction transformation, a mathematical relationship that shows how one can go from a more recent level of physics down to, say, Newtonian. For example, the Lorentz transformation is a classic example which takes the non-linear and non-Cartesian system of Relativity and shows that Newton's equations still hold in the mesoscale of the regular world. The Schwarzian derivative, which is a linear fractional transformation of a complex function, is an example in complexity theory.

Each step or revolution in physics seemed to be preceded by a major shift in philosophical speculations. Perhaps unification theory in physics is simply waiting for us philosophers to provide the key speculation.

Whitehead's philosophy is only the beginning of our work. If a unified theory of physics is possible, it is going to require further progression of metaphysics and our current work in process philosophy. We have shown that characteristic randomness implies a basic instability in the foundations of metaphysics and that information, process and randomness are crucial points within the argument. Perhaps a new type of information process metaphysics is the key to bridging process philosophy with a modern mathematical-scientific understanding of the world. These possibilities give credence to the continued exploration of process thinking, especially

at the foundational level of mathematics, and more importantly, at the foundational level of metaphysics itself.